PHYSICAL REVIEW B **80**, 041313(R) (2009)

Optimal electron entangler and single-electron source at low temperatures

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(Received 3 July 2009; published 29 July 2009)

We show that the equilibrium noise can be dramatically suppressed in the case of a tunnel junction with modulated (time-dependent) transparency. We demonstrate how such a contact could be used either as an optimal electron entangler or as a single-electron source with suppressed equilibrium noise at low temperatures.

DOI: 10.1103/PhysRevB.80.041313 PACS number(s): 73.23.−b, 03.65.Ud, 72.70.+m

The controlled production, manipulation, and detection of entangled particles are central to quantum computation. Considerable progress has been made on the manipulation of several qubits in quantum optics.[1] Given the controllability of device parameters and the rapid and coherent transport through mesoscopic contacts, one expects that electrons could also be used to process quantum information.[2,3]

Recently, Neder *et al.*[4] reported the observation of interference between two electrons emitted from independent sources in an electronic Mach-Zehnder interferometer. However, it was not possible to demonstrate quasiparticle entanglement conclusively owing to the dephasing effects and thermal noise.[5] Another breakthrough in "electronic optics" was the development of a single-electron source on demand with subnanosecond time resolution by Fève *et al.*[6] They used the coupling of the localized level of a quantum dot to a ballistic conductor to obtain a sequence of single-electron pulses. A proposal for the operation of this device with minimal noise has also been published.[7]

Electron entanglement could be demonstrated in measurements of the zero-frequency current cross-correlation function of a tunnel junction,[3] where it would contribute to the shot noise associated with the pumping cycle. This contribution would need to be distinguished from the contribution of "classical" thermal charge noise, $\langle\langle Q^2\rangle\rangle_{th}\propto Tt_f/h$, and equilibrium noise. Here T is temperature, t_f is the measurement time, and h is Planck's constant. For experimentally available temperatures $T=10$ mK, this requires operating at high (GHz) frequencies ($t_f \ll 10^{-8}$ s), which are now experimentally[6,8] accessible. However, when $Tt_f/h\ll 1$, the problem of equilibrium noise associated with the starting and ending of measurement arises.[9] In general the connection-disconnection process leads to a logarithmically divergent equilibrium charge noise $\langle\langle Q^2\rangle\rangle \propto \log(t_f\epsilon_F/h)$, where ϵ_F is the Fermi energy of the electron gas.[9] Even with a measuring time $t_f=10^{-9}$ s the equilibrium charge noise is of order 1. This would be a factor of 2 larger than the shot noise associated with the generation of an entangled electron-hole pair, which cannot exceed 1/2 [the maximum theoretical entanglement is 50% (Ref. 10)].

At first glance the logarithmically divergent equilibrium noise seems unavoidable because it reflects the fluctuation in the number of fermions in one-dimensional wire crossing any point during a measurement time interval. However, we demonstrate that choosing the measurement procedure appropriately can dramatically reduce the logarithmic equilibrium noise making entanglers and sources for ultracold single electrons a realistic prospect.

We consider a device consisting of two one-dimensional wires coupled by a tunnel junction with tunable transmission and reflection amplitudes A and B [Fig. 1(a)] at zero temperature. We show that tuning the transparency amplitude (assumed real) to take the time-dependent form,

$$A = \operatorname{Im}\frac{t-t_0-i\tau_0}{t-t_0+i\tau_0}, \tag{1}$$

where the parameters t_0 and τ_0 describe the position and the width of a pulse, leads to the excitation of a single spin-entangled particle-hole pair, with (τ_0-independent) probability 1/2 to find the particle in one lead and the hole in the other. Experimentally, the transparency of a quantum point contact (QPC) can be controlled by the gate voltage V_G. The transmission coefficient $|A|^2$ for a single-channel QPC was shown to be a smooth and easily controlled function of the gate voltage.[11,12] Although the protocol could be implemented to generate spin or orbital entanglement, e.g., the one investigated experimentally,[4] we concentrate on how the equilibrium noise associated with the measurement procedure can be completely eliminated in the simplest spin-entangled particle source proposed by Beenakker *et al.*[10]

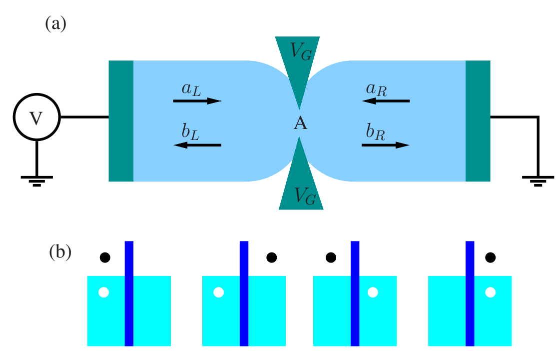

FIG. 1. (Color online) Charge transport through a quantum point contact. (a) Tunnel junction with the transparency amplitude A controlled by a gate voltage V_G. The unbiased contact ($V=0$) can be used as an optimal entangler. With a Lorentzian bias voltage profile, $V(t)$, the device could be operated as a single-electron source (see text). (b) Outcomes of applying pulse (1) to the barrier of the unbiased tunnel junction. A hole (white) and an electron (black) occupy states in both leads with probability 1/2.

The noise in the charge transferred across a point contact and the degree of entanglement of states on either side of the contact can be controlled by the time-dependence of the barrier transparency. The noise in the charge transferred across the barrier $\langle\langle Q^2\rangle\rangle=\int dtdt'\langle\Delta I(t)\Delta I(t')\rangle$, where $\Delta I=I-\langle I\rangle$, where I is the current operator, will be given by[13]

$$\langle\langle Q^2\rangle\rangle=\int dtdt'\{2A(t)^2A(t')^2n_R(t,t')(1-n_R(t',t))+A(t)B(t)A(t')B(t')[n_L(t,t')(1-n_R(t',t))+(1-n_L(t,t'))n_R(t',t)]\}. \tag{2}$$

Here $n_{L(R)}$ is the density matrix of incoming states in the left (right) lead. With no bias voltage and at zero temperature, $n_{L,R}=n_0=i/[2\pi(t-t'+i0)]$, where n_0 is the Fourier transform of the Fermi-Dirac distribution. If the transparency amplitude A switches abruptly, $A\neq 0$ for $t\in[0,t_f]$ and $A=0$ otherwise, one obtains the well-known logarithmically divergent result for the zero bias ($V=0$) noise $\langle\langle Q^2\rangle\rangle_0=\frac{A^2}{\pi^2}\log(t_f\epsilon_F/h)$. The logarithmic term is present in general and not just for sudden switching. It was found, for example, for the case of a Gaussian profile.[14] However, as we show below, in the particular case where $A(t)$ is chosen in accordance with Eq. (1), the term is absent and $\langle\langle Q^2\rangle\rangle_0=1/2$. We show how this result can be used together with the spin degree of freedom of electrons to define an optimum entanglement protocol.

The statistics of the transferred charge are encoded in the characteristic function, $\chi(\lambda)$,[15]

$$\chi(\lambda)=\sum_{n=-\infty}^{+\infty}P_ne^{in\lambda}. \tag{3}$$

Here P_n is the probability of n particles being transmitted across a barrier. A variety of approaches have been used to study $\chi(\lambda)$, with the majority focused on voltage-biased contacts.[15–21] We find a mapping between the problem of the biased leads and that of the time-dependent barrier transmission. Our formulation of the problem enables us to solve for $\chi(\lambda)$ analytically in the unbiased contact and to compute numerically the characteristic function for a time-dependent transmission amplitude in the presence of bias voltages.

For the tunnel junction of Fig. 1(a), the annihilation operators of the incoming states $a_{L(R)}$ are related to the outgoing states $b_{L(R)}$ at the left (right) lead via

$$\begin{pmatrix}b_L\\ b_R\end{pmatrix}=S\begin{pmatrix}a_L\\ a_R\end{pmatrix}\quad\text{with}\quad S=\begin{pmatrix}B & A\\ -A & B\end{pmatrix}, \tag{4}$$

where S is the scattering matrix. In general, one can use the multienergy Floquet scattering matrix,[14] which requires the solution to the full time-dependent scattering problem. But in the low-frequency limit, when the scattering matrix changes slowly on the scale of the Wigner delay time t_W, one can use stationary scattering matrix (4) instead (adiabatic approximation).[17] Far from resonances the Wigner delay time is small, which enables us to use the adiabatic approximation for the frequencies $\nu\ll\sim 1$ THz.[22] We can also neglect the Coulomb interaction as there are no Coulomb blockade effects to consider in a tunnel junction at the frequencies $\nu\ll\sim 1$ THz. In the (time-independent) basis in which the scattering matrix is diagonal, the incoming states $\tilde{a}_1=\frac{1}{\sqrt{2}}(a_L-ia_R)$ and $\tilde{a}_2=\frac{1}{\sqrt{2}}(a_L+ia_R)$. In this basis $S_{11}=e^{i\phi(t)}$, $S_{22}=e^{-i\phi(t)}$, and $S_{12}=S_{21}=0$, where $e^{i\phi(t)}=B(t)+iA(t)$, and the outgoing states are $b_L=\frac{1}{\sqrt{2}}(\tilde{a}_1e^{i\phi}+\tilde{a}_2e^{-i\phi})$ and $b_R=\frac{i}{\sqrt{2}}(\tilde{a}_1e^{i\phi}-\tilde{a}_2e^{-i\phi})$. This enables us to calculate the characteristic function geometrically.[21]

The statistics of the charge transfer across the barrier induced by the time dependence of the scattering phase, $\phi(t)$, can be computed for systems at $T=0$ from the eigenvalues of the matrix $h_1h_2=e^{i\phi}he^{-i\phi}e^{-i\phi}he^{i\phi}$ where $h=2n_0-1$ and n_0 is the ground state density matrix.[19] The eigenvalues of h_1h_2 are identical to those of

$$\tilde{h}h\equiv e^{i2\phi}he^{-2i\phi}h. \tag{5}$$

Eigenvalues different from 1 occur in pairs, $e^{\pm i\alpha_j}$, with the index j labeling separate and independent excitations of the system. (Eigenvalues equal to 1 relate to states, which are either occupied or empty in both channels and which, therefore, do not contribute to the charge transfer.) The probability of exciting a particle-hole pair in a given state is given by $\sin^2\alpha_j/2$.[21] When transforming back to the basis of left and right leads, this gives the probability for finding no additional charge in the right lead as a result of the excitation in the state j as $P_0=\cos^2\frac{\alpha_j}{2}+\frac{1}{2}\sin^2\frac{\alpha_j}{2}$, while the probabilities for finding an additional particle or hole are: $P_{\pm1}=\frac{1}{4}\sin^2\frac{\alpha_j}{2}$. Characteristic function (3) is the product of the results for the independent states,

$$\chi(\lambda)=\prod_j\left(1+\frac{1}{4}\sin^2\frac{\alpha_j}{2}(e^{i\lambda}+e^{-i\lambda}-2)\right). \tag{6}$$

The operator $\tilde{h}h$ in Eq. (5) is formally the same as the one that appears in the treatment of the much studied problem of a quantum contact with a bias. A time-dependent voltage, $V(t)$, is applied between the leads and the barrier is described by time-independent transmission and reflection amplitudes, A and B.[9,15,16,19,21] There, the angles α_j are found by diagonalizing $e^{i\psi(t)}he^{-i\psi(t)}h$, where $\psi=e/\hbar\int^tV(t')dt'$ is the Faraday flux.[19] Direct comparison with Eq. (5) shows that the problem of the unbiased quantum contact, with time-dependent scatterer, is equivalent to the problem of a voltage biased quantum contact, with time-independent scatterer, if $\psi\rightarrow 2\phi$, $|A|^2\rightarrow 1/2$, and $|B|^2\rightarrow 1/2$. To see the power of this mapping, consider the case of a periodically modulated transparency $A=\sin(\omega t)$ and $B=\cos(\omega t)$. The solution to the corresponding problem for the contact under constant bias is known[15] and gives $\chi(\lambda)=[(1+\cos\lambda)/2]^{\omega t_f/2\pi}$—a result previously obtained using a much longer calculation by Andreev and Kamenev.[23]

Particular attention has been paid in the case of the biased contact to quantized Lorentzian voltage pulses applied to one electrode $V(t)=\frac{-2\tau_0}{(t-t_0)^2+\tau_0^2}$ when

$$e^{i\psi(t)} = \frac{t - t_0 - i\tau_0}{t - t_0 + i\tau_0}. \quad (7)$$

The pulse excites exactly one particle or hole depending on the polarity of the device.[16,24] The corresponding eigenvalue of $\tilde{h}h$ is -1.

The analog of the quantized Lorentzian pulses in the case of the biased junction is the key to optimizing the entanglement between states in the case of a time-dependent barrier. If we choose the barrier modulation phase $e^{i\phi(t)}=B(t)+iA(t)$ to be given by Eq. (7), this corresponds to choosing the transparency amplitude to be given by Eq. (1). In the language of the biased lead case, this gives rise to two so-called unidirectional events of opposite polarity in independent channels 1 and 2.

There are then two eigenvalues of $\tilde{h}h$ different from 1. Both are equal to -1, so $\alpha=\pi$.[21] From Eq. (6) we then obtain $\chi(\lambda)=\frac{(1+\cos\lambda)}{2}$. The four possible outcomes of applying pulse (1) to the barrier are illustrated in Fig. 1(b) and all occur with probability 1/4.

The particle and the hole in Fig. 1(b) are entangled. However, this entanglement cannot be revealed by measurements because of particle number conservation.[25] In order to create "useful" entangled states, which can be measured with the Bell procedure, we should consider entanglement in, for example, the spin degree of freedom of the particles.

To calculate the available entanglement entropy, one should use the super-selection rules, which account for particle number conservation.[26–28] Let a quantum state, with N particles distributed between Alice and Bob (left and right leads), be described by a wave function $|\Psi_{AB}\rangle$ and let $|\Psi^n_{AB}\rangle$ be the wave function projected onto a subspace of fixed local particle number n for Alice and $N-n$ for Bob. The available entropy is: $S_{avl}=\Sigma_n P_n S_n$, where P_n is the probability $P_n=\langle\Psi^n_{AB}|\Psi^n_{AB}\rangle/\langle\Psi_{AB}|\Psi_{AB}\rangle$ and S_n is the standard entanglement entropy corresponding to the configuration n. The probability to find a particle in the left lead and a hole in the right lead or vice versa is 1/4. The entanglement entropy of the configurations containing a fully entangled Bell pair is $S_n=1$. Thus we find $S_{avl}=1/2$. This corresponds to an entangler with 50% efficiency, which is the optimal value. The entangler proposed originally[10] has the same efficiency but still contains the equilibrium noise, which in our proposal is completely suppressed.

Now we show how a biased quantum point contact with tunable reflection and transmission amplitudes A and B could be operated as a single-electron source. It is known[16,24] that a Lorentzian voltage pulse $V(t)$ applied between the leads of a quantum point contact excites a single electron (or hole), provided the Faraday flux, $\psi=e/\hbar\int^t V(t')dt'$, is an integer multiple of 2π [see Eq. (7)]. However, if the barrier transparency is turned on abruptly, the logarithmically divergent equilibrium noise emerges and makes measurement of a single unidirectional event problematic.

To suppress the equilibrium noise, we propose modulating the barrier smoothly by applying a pulse sequence to the barrier,

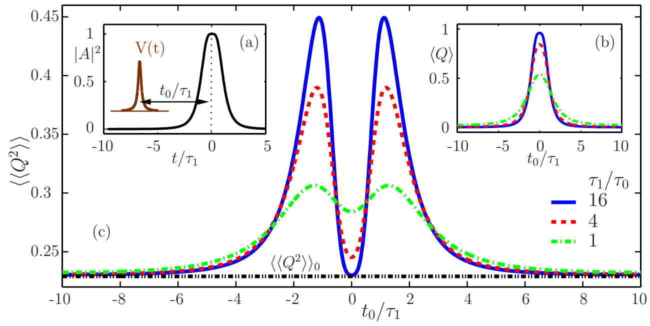

FIG. 2. (Color online) Single unidirectional event in a quantum contact with tunable transparency. (a) Transparency of the barrier, $|A(t)|^2$, as a function of time [see Eq. (8)]. Also shown is the quantized voltage pulse, $V(t)$, applied to the left lead; (b) average charge transfer through the barrier, $\langle Q\rangle$, as a function of t_0/τ_1, where t_0 is the separation between the maxima of $V(t)$ and $|A(t)|^2$ [see Fig. 2(a)]. The maximum of $\langle Q\rangle$ occurs when $t_0=0$ where $|A|^2$ approaches 1; it grows as the width of the voltage pulse, τ_0, is reduced and is close to 1 for $\tau_0=\tau_1/16$. (c) Noise, $\langle\langle Q^2\rangle\rangle$, in a biased contact with transparency given by Eq. (8) as a function of t_0/τ_1. The minimum of $\langle\langle Q^2\rangle\rangle$ occurs for $t_0=0$ and, for a narrow voltage pulse ($\tau_0=\tau_1/16$), it approaches the limit set by the barrier modulation: $\langle\langle Q^2\rangle\rangle_0$ (horizontal dashed line). The maxima in the noise occur when $t_0/\tau_1\approx 1$ and $|A|^2\approx 1/2$.

$$e^{i\phi} = B + iA = \left(\frac{t + t_1/2 - i\tau_1}{t + t_1/2 + i\tau_1}\right)\left(\frac{t - t_1/2 + i\tau_1}{t - t_1/2 - i\tau_1}\right). \quad (8)$$

We choose $\tau_1=t_1(\frac{1}{2}+\frac{1}{\sqrt{2}})$. This choice gives a transmission amplitude, which does not change sign and goes through a single maximum at $t=0$, with $|A(0)|^2=1$ [see Fig. 2(a)]. In addition, a quantized bias voltage pulse is applied between the leads so that $n_R=n_0$ and $n_L=e^{i\psi}n_0e^{-i\psi}$, with Faraday flux $\psi(t)$ given by Eq. (7). If we choose a pulse, which is narrow in time ($\tau_0\ll\tau_1$), we should expect on the basis of Eq. (2) that

$$\langle\langle Q^2\rangle\rangle \approx \langle\langle Q^2\rangle\rangle_0 + |A(t_0)|^2(1-|A(t_0)|^2). \quad (9)$$

The first term describes the zero bias noise associated with the opening of the contact and the second gives the shot noise for an open contact.[9] For $t_0=0$, the second term can be made to vanish so that for this choice of τ_1/t_1, we should expect that $\langle\langle Q^2\rangle\rangle\approx\langle\langle Q^2\rangle\rangle_0\approx 1/4$.

In the presence of a time-dependent barrier and bias, the analysis of the characteristic function, $\chi(\lambda)$, is slightly more involved but can be easily computed numerically. The characteristic function in the general case of quantum pumping can be expressed in terms of the eigenvalues of the single-particle density matrix of outgoing states in, say, the right lead $n_{out}=\langle b_R^\dagger b_R\rangle=B^*n_0B+A^*\tilde{n}A$. Here $\tilde{n}=e^{i\psi}n_0e^{-i\psi}$. Consider first the case of a single-particle excitation in the outgoing states. The eigenvalues of the density matrix in the right lead are 1 (for the states below the Fermi level), 0 (for the states above the Fermi level not occupied by the excitation), and n_j for the state affected by the excitation. Characteristic function (3) is then $\chi_p(\lambda)=1-n_j+e^{i\lambda}n_j$. For a single hole in the outgoing states we find $\chi_h(\lambda)=n_j+e^{-i\lambda}(1-n_j)$. These two formulae can be combined to give $\chi(\lambda)=e^{i(Q_1-n_j)\lambda}(1-n_j$

$+e^{i\lambda}n_j$). (The transferred charge $Q_1=n_j$ for the particle excitation and $Q_1=n_j-1$ for the hole excitation.) In the general case the density matrix is Hermitian and therefore possesses an orthonormal eigenbasis. The excitations labeled by j are therefore independent, and we arrive at the formula of[29] $\chi(\lambda)=e^{i\lambda Q}\prod_j e^{-i\lambda n_j}[1+(e^{i\lambda}-1)n_j]$ where Q is the total transferred charge.

For the case of pulse (8) applied to the barrier together with bias pulse (7) applied to the left lead, we have diagonalized the density matrix n_{out} numerically. In Fig. 2(c) we present results for the noise $\langle\langle Q^2\rangle\rangle$ in the system as a function of the separation t_0 between the bias pulse and the barrier pulses. The maximum values of the noise occur when $t_0\approx\tau_1$ and the transparency coefficient is almost 1/2. This is the regime where the barrier is acting as a 50% beam splitter. The minimum of $\langle\langle Q^2\rangle\rangle$ corresponds to $t_0=0$, when $|A|^2=1$. At $t_0=0$ the transferred charge also reaches its maximum [see Fig. 2(b)]. If the bias pulse is narrow compared to the barrier pulse ($\tau_0\ll\tau_1$), the major contribution to the noise at the minimum is very close to what was expected from Eq. (9), namely, $\langle\langle Q^2\rangle\rangle_0$, and the transferred charge approaches 1. This is the regime that could be used as a single-electron source on demand at ultralow temperatures.

In summary, equilibrium noise is a quantum effect present even at zero temperature, which is associated with multiple low-energy particle-hole excitations. At experimentally accessible temperatures and frequencies it would obscure the observation of entanglement (we have estimated that the noise is of order twice the theoretical maximum signal). We have described a protocol that eliminates this equilibrium noise in a tunnel junction operated as an electronic entangler and shows that the theoretical maximum could be achieved. Our protocol requires the use of a tailored time dependence for the transparency of a tunnel barrier. This excites a single particle-hole pair as a result of a many-body effect similar to the one considered by Keeling *et al.*,[24] which was for the case of a bias voltage applied across the contact. We also show how a combination of time dependence for barrier transmission (8) and for the bias voltage between leads can reduce the equilibrium noise for the device when operated as a single-electron source.

The work was supported by EPSRC-GB (Contract No. EP/D065135/1).

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